Pharmaceutical statistics

2025-2024

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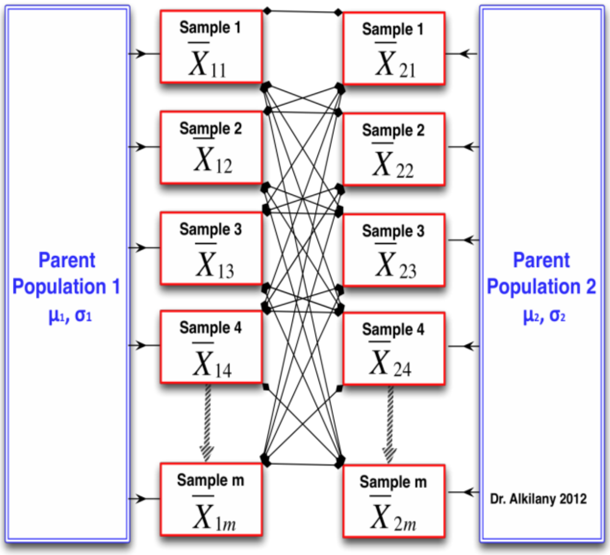
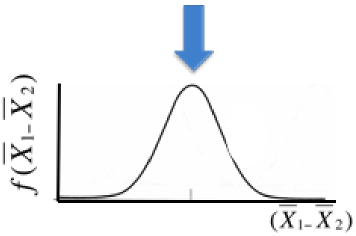
ARKAN
ACADEMY

Distributions of the Difference Between Two Sample Means ∆

- **We have 2 populations 1 & 2 with known μ and σ**
  - We take all possible samples from each population and calculated the sample mean ($\bar{x}$).
  - Then we took all possible difference between the sample means ($\bar{x}_{11} - \bar{x}_{21}$) and ($\bar{x}_{12} - \bar{x}_{21}$) and vice versa ($\bar{x}_{21} - \bar{x}_{11}$)
  - Then we can construct a distribution using (($\bar{x}_{11} - \bar{x}_{21}$) and f ($\bar{x}_{11} - \bar{x}_{21}$)).
  - The mean of the difference between two sample means **μ ($\bar{x}_1 - \bar{x}_2$)** is equal to ($\mu_1 - \mu_2$).
  - **σ ($\bar{x}_1 - \bar{x}_2$)** or **SE** is calculated as: $\sqrt{\frac{\sigma 1^2}{n1} + \frac{\sigma 2^2}{n2}}$





★ **Example 1:**

1. **Suppose that we have the following 2 population each with 3 values**

   Pop 1: 6, 8, 10 (μ_1=8, σ_1=1.63)

   Pop 2: 2,4, 6 (μ_2=4, σ_2=1.63)

Table 1.

All possible sample means ($\overline{X1}$) from pop. 1. n =2

	6	8	10
6	6	7	8
8	7	8	9
10	8	9	10

Sample X11 (6,6)

Sample X14 (8,6) **Sample X19 (10, 10)**

Table 2.

All possible sample means ($\overline{X2}$) frompop.2. n = 2

	2	4	6
2	2	3	4
4	3	4	5
6	4	5	6

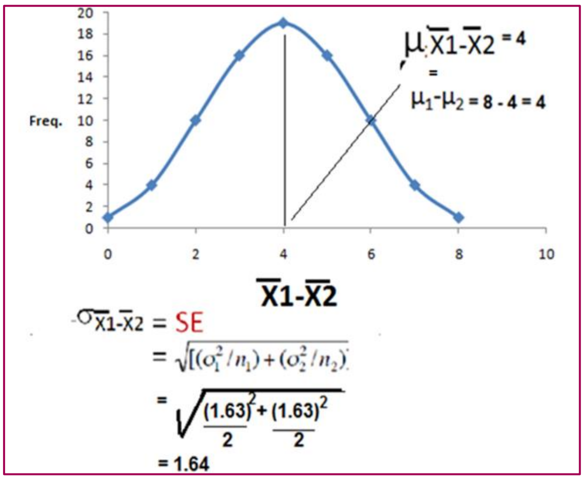
Sample X26 (4,6)

$\overline{X1}$	$\overline{X2}$	$\overline{X1}$-$\overline{X2}$	$\overline{X1}$	$\overline{X2}$	$\overline{X1}$-$\overline{X2}$	$\overline{X1}$	$\overline{X2}$	$\overline{X1}$-$\overline{X2}$
6	2	4	7	2	5	8	2	6
6	3	3	7	3	4	8	3	5
6	4	2	7	4	3	8	4	4
6	3	3	7	3	4	8	3	5
6	4	2	7	4	3	8	4	4
6	5	1	7	5	2	8	5	3
6	4	2	7	4	3	8	4	4
6	5	1	7	5	2	8	5	3
6	6	0	7	6	1	8	6	2
7	2	5	8	2	6	9	2	7
7	3	4	8	3	5	9	3	6
7	4	3	8	4	4	9	4	5
7	3	4	8	3	5	9	3	6
7	4	3	8	4	4	9	4	5
7	5	2	8	5	3	9	5	4
7	4	3	8	4	4	9	4	5
7	5	2	8	5	3	9	5	4
7	6	1	8	6	2	9	6	3
8	2	6	9	2	7	10	2	8
8	3	5	9	3	6	10	3	7
8	4	4	9	4	5	10	4	6
8	3	5	9	3	6	10	3	7
8	4	4	9	4	5	10	4	6
8	5	3	9	5	4	10	5	5
8	4	4	9	4	5	10	4	6
8	5	3	9	5	4	10	5	5
8	6	2	9	6	3	10	6	4

- ✓ Average of $\bar{x}1 - \bar{x}2$ (μ ($\bar{x}_1 - \bar{x}_2$) = 4
- ✓ St. deviation of $\bar{x}1 - \bar{x}2$ ($\sigma\bar{x}_1 - \bar{x}_2$) = 1.64

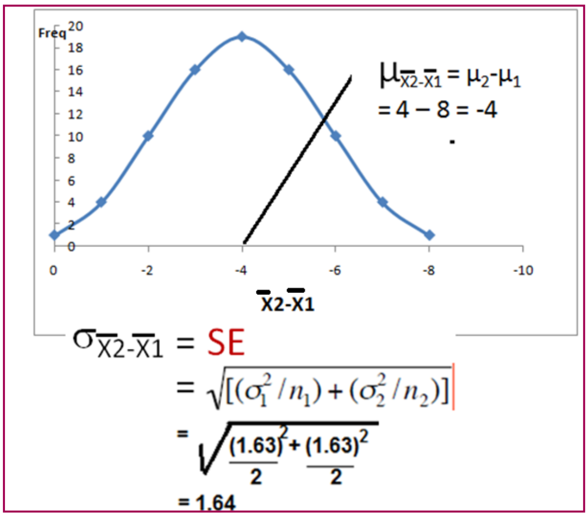
- Summary of Data in Table 3 as frequency distribution Table and distribution of the difference between sample means based on x̅1- x̅2.

$\overline{X}1-\overline{X}2$	Freq
0	1
1	4
2	10
3	16
4	19
5	16
6	10
7	4
8	1

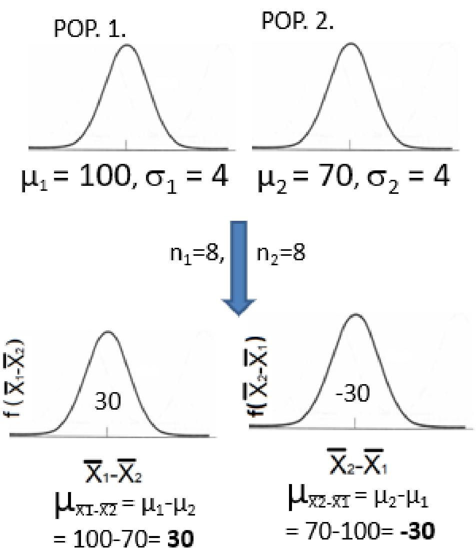


- Frequency distribution Table and distribution of the difference between sample means for example 1 based on x̅2- x̅1.

$\overline{X}2-\overline{X}1$	Freq
-8	1
-7	4
-6	10
-5	16
-4	19
-3	16
-2	10
-1	4
0	1



- **σ (x̅₁- x̅₂) = σ (x̅₂- x̅₁)**
- **μ (x̅₁- x̅₂) = - μ (x̅₂ - x̅₁)**



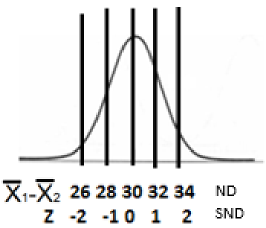
★ **Example 2:**

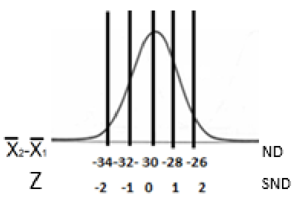
Z = (x̅1- x̅2)-(μ1-μ2)/SE

Z_{26} = (26)-(30)/2= -2

Z = (x̅2- x̅1)-(μ2-μ1)/SE

Z_{-34} = (-34)-(-30)/2= -2





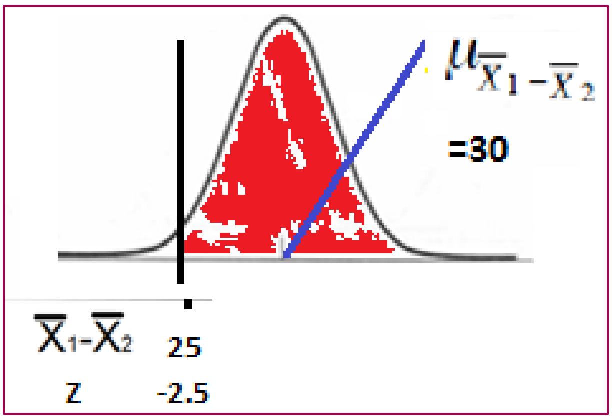
$\sigma_{\overline{X}1-\overline{X}2} = \sigma_{\overline{X}2-\overline{X}1}$ = SE

$= \sqrt{[(\sigma_1^2/n_1)+(\sigma_2^2/n_2)]}$

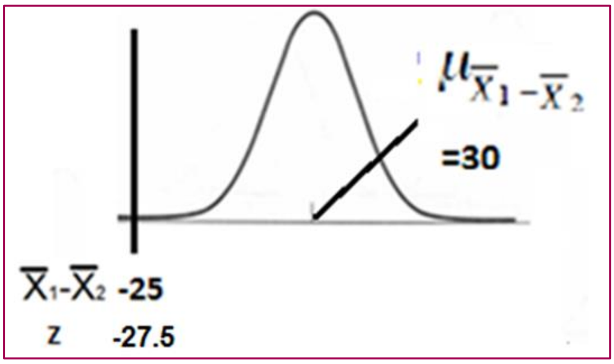
$= \sqrt{16/8 + 16/8)} = 2$

- ✓ ***Q1.*** If a random sample was taken from each population, what is the probability, the difference between the means ($\bar{x}1-\bar{x}2$) is between 28 and 32.
  Answer: $P(28 \leq \bar{x}1-\bar{x}2 \leq 32) = P(-1 \leq Z \leq +1) = 0.68$
- ✓ ***Q2.*** If a random sample was taken from each population, what is the probability the difference between the means ($\bar{x}2-\bar{x}1$) is between -32 and -28.
  Answer: $P(-32 \leq \bar{x}2-\bar{x}1 \leq -28) = P(-1 \leq Z \leq +1) = 0.68$
- ✓ ***Q3.*** If a random sample was taken from each population, what is the probability the difference between the means ($\bar{x}1-\bar{x}2$) is between 26 and 34.
  Answer: $P(26 \leq \bar{x}1-\bar{x}2 \leq 34) = P(-2 \leq Z \leq +2) = 0.95$
- ✓ ***Q4.*** If a random sample was taken from each population, what is the probability the difference between the means ($\bar{x}2-\bar{x}1$) is between -34 and -26.
  Answer: $P(-34 \leq \bar{x}2-\bar{x}1 \leq -26) = P(-2 \leq Z \leq +2) = 0.95$
- ✓ ***Q5.*** A sample of population 1 is at least 25 units higher than that of population 2.

$Z_{25} = (\bar{x}1-\bar{x}2) - (\mu_1-\mu_2)/SE$
$Z_{25} = (25)-(30)/2 = -2.5$
$P((\bar{x}1-\bar{x}2) \geq 25) = P(Z \geq -2.5)$
$= 1-P(Z \leq -2.5) = 1 - 0.0048 = 0.9952$



- ✓ ***Q6.*** A sample of population1 is at least 25 units smaller than that of population 2.
  $Z_{-25} = (\bar{x}1-\bar{x}2) - (\mu_1-\mu_2)/SE$
  $Z_{-25} = (-25) - (30)/2 = -27.5$
  $P((\bar{x}1-\bar{x}2) \leq -25) = P(Z \leq -27.5) = 0$



- ✓ ***Q7.*** The difference between the samples means is at least 25.
  $P(\bar{x}1-\bar{x}2 \geq 25) + P(\bar{x}1-\bar{x}2 \leq -25) = 0.9952 + 0 = 0.9952$

★ **Example 3:**

In two populations: Population 1 has experienced some condition that is associated with mental retardation. The second population has not experienced these conditions. The distribution of intelligence scores in each of the two populations (1 and 2) is believed to be normally distributed and equal for both with standard deviation of **20**.

A sample of **15** individuals from each population were withdrawn, compute the probability of the difference between two means to be equal or larger than 13?

$$\mu_{\overline{X}_1-\overline{X}_2} = \mu_{\overline{X}_1} - \mu_{\overline{X}_2} = \mu_1 - \mu_2 = 0$$

$$\sigma_{\overline{X}_1-\overline{X}_2} = \sqrt{[(\sigma_1^2/n_1)+(\sigma_2^2/n_2)]} = SE(\Delta)$$
$$= \sqrt{[(20^2/15)+(20^2/15)]}$$
$$= 7.3$$

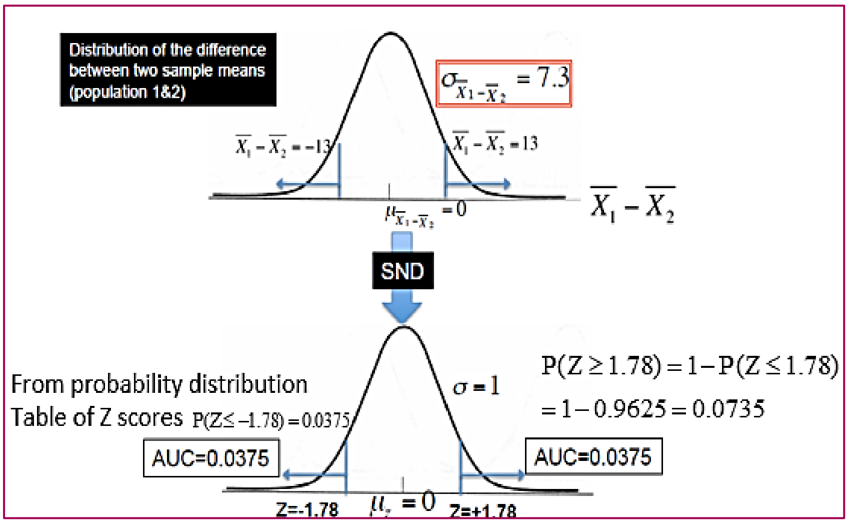
First case : $\overline{x}_1 - \overline{x}_2 \geq 13$

$$z_{13} = \frac{(\overline{x}_1-\overline{x}_2)-\mu_{\overline{X}_1-\overline{X}_2}}{\sigma_{\overline{X}_1-\overline{X}_2}} = \frac{13}{7.3} = +1.78$$

Second case : $\overline{x}_2 - \overline{x}_1 \geq 13 .... > \overline{x}_1 - \overline{x}_2 \leq -13$

$$z_{-13} = \frac{(\overline{x}_1-\overline{x}_2)-\mu_{\overline{X}_1-\overline{X}_2}}{\sigma_{\overline{X}_1-\overline{X}_2}} = \frac{-13}{7.3} = -1.78$$

✓ $P(\overline{X}_1-\overline{X}_2)\geq 13 + P(\overline{X}_1-\overline{X}_2)\leq -13$
$= P(Z\geq 1.78)+P(Z\leq -1.78)$
$= 1-P(Z\leq 1.78)+P(Z\leq -1.78) =$
$1-0.96250+0.0375 = 0.0375+0.0375$
$= 0.075$



★ **Example 4:**

Population A: μ_A =45 min, σ_A=15 min
Population B: μ_B =43 min, σ_B = 20 min
If we select 35 variables from pop A (sample A) and 40 variables from pop B (sample B), what is the probability that the means for samples A&B will differ by 5 minutes or more?

$\mu_A = 45, \sigma_A = 15, n_A = 35$
$\mu_B = 43, \sigma_B = 20, n_B = 40$
$\mu_{\overline{X}_A-\overline{X}_B} = \mu_A - \mu_B = 2$
$\sigma_{\overline{X}_A-\overline{X}_B} = \sqrt{[(\sigma_A^2/n_A)+(\sigma_B^2/n_B)]} = 4.0$

first case :
$\overline{X}_A - \overline{X}_B \geq 5$
$z_5 = \frac{5-2}{4.0} = 0.75$
$P(z\geq 0.75) = 1-P(z\leq 0.75)$
$= 0.23$

second case :
$\overline{X}_B - \overline{X}_A \geq 5, then: \overline{X}_A - \overline{X}_B \leq -5$
$z_{-5} = \frac{-5-2}{4.0} = -1.75$
$P(z\leq -1.75) = 0.04$

Overall probability=0.23+0.04=0.27

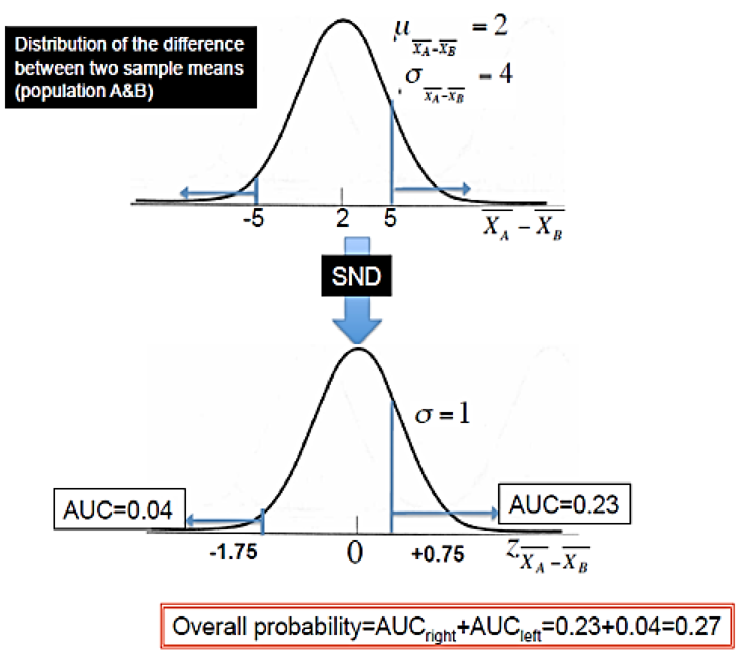
✓ $P(\overline{X}_1 - \overline{X}_2) \geq 5 + P(\overline{X}_1 - \overline{X}_2) \leq -5$

$= P(Z \geq 0.75) + P(Z \leq -1.75)$

$= 1 - P(Z \leq 0.75) + P(Z \leq -1.75) =$

$1 - 0.7673 + 0.0418 = 0.2327 + 0.0418$

$= 0.27$



★ **Example 5:**

Population A: μ_A =45 min, σ_A=15 min
Population B: μ_B =30 min, σ_B=20 min

If we select 35 variables from pop A (sample A) and 40 variables from pop B (sample B), what is the probability that the means for samples A&B will differ by 20 minutes or more?

$\mu_A = 45, \sigma_A = 15, n_A = 35$

$\mu_B = 30, \sigma_B = 20, n_B = 40$

$\mu_{\overline{X_A}-\overline{X_B}} = \mu_A - \mu_B = 15$

$\sigma_{\overline{X_A}-\overline{X_B}} = \sqrt{[(\sigma_A^2 / n_A) + (\sigma_B^2 / n_B)]} = 4.05$

first case:

$\overline{X_A} - \overline{X_B} \geq 20$

$z_{20} = \frac{20 - 15}{4.05} = 1.23$

$P(z \geq 1.23) = 0.11$

$\mu_A = 45, \sigma_A = 15, n_A = 35$

$\mu_B = 30, \sigma_B = 20, n_B = 40$

$\mu_{\overline{X_A}-\overline{X_B}} = \mu_A - \mu_B = 15$

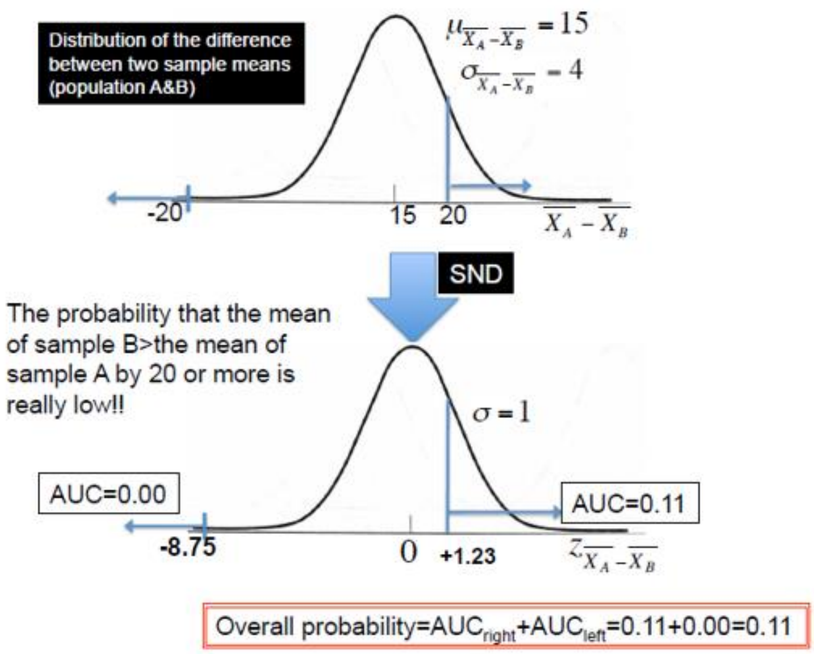
$\sigma_{\overline{X_A}-\overline{X_B}} = \sqrt{[(\sigma_A^2 / n_A) + (\sigma_B^2 / n_B)]} = 4.05$

second case:

$\overline{X_B} - \overline{X_A} \geq 20, then: \overline{X_A} - \overline{X_B} \leq -20$

$z_{-20} = \frac{-20 - 15}{4.05} = -8.75$

$P(z \leq -8.75) = zero$



★ **Example 6:**

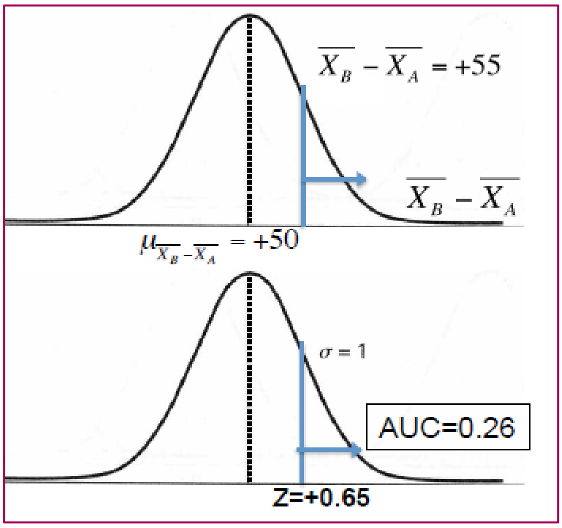
The capsule weight for two hard-gelatin capsule batches A&B are normally distributed with the following parameters:

Batch A: μ=250 mg; σ =25 mg

Batch B: μ=300 mg; σ =35 mg

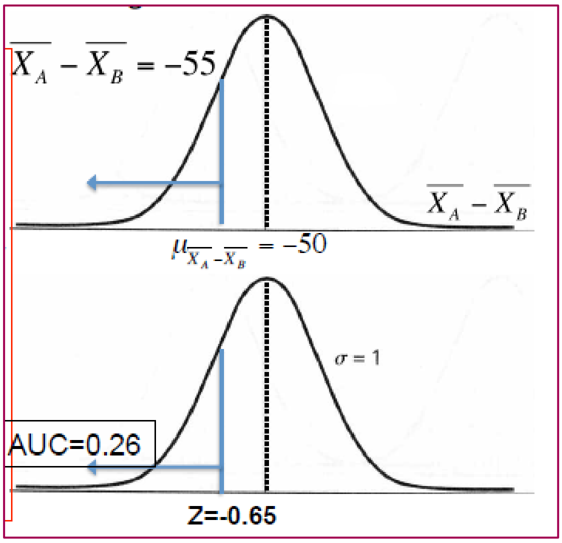
If we withdraw a random sample from A (30 capsules) and a random sample from B (40 capsules), what is the probability that the mean of sample B is larger than the mean of sample A by at least 55 mg?

$$\mu_A = 250, \sigma_A = 25, n_A = 30$$
$$\mu_B = 300, \sigma_B = 35, n_B = 40$$
$$\mu_{\overline{X}_B - \overline{X}_A} = \mu_B - \mu_A = +50$$
$$\sigma_{\overline{X}_B - \overline{X}_A} = \sqrt{[(\sigma_B^2 / n_B) + (\sigma_A^2 / n_A)]} = 7.17$$
$$\overline{X}_B - \overline{X}_A \geq 55$$
$$z_{55} = \frac{55 - 50}{7.71} = +0.65$$



➢ **Another way to solve it:**

$$\mu_A = 250, \sigma_A = 25, n_A = 30$$
$$\mu_B = 300, \sigma_B = 35, n_B = 40$$
$$\mu_{\overline{X}_A - \overline{X}_B} = \mu_A - \mu_B = -50$$
$$\sigma_{\overline{X}_A - \overline{X}_B} = \sqrt{[(\sigma_A^2 / n_A) + (\sigma_B^2 / n_B)]} = 7.17$$
$$\overline{X}_B - \overline{X}_A \geq 55$$
$$\overline{X}_A - \overline{X}_B \leq -55$$
$$z_{-55} = \frac{-55 - (-50)}{7.71} = -0.65$$



CASE OF A-B

$$\mu_A = 250, \sigma_A = 25, n_A = 30$$
$$\mu_B = 300, \sigma_B = 35, n_B = 40$$
$$\mu_{\overline{X}_A - \overline{X}_B} = \mu_A - \mu_B = -50$$
$$\sigma_{\overline{X}_A - \overline{X}_B} = \sqrt{[(\sigma_A^2 / n_A) + (\sigma_B^2 / n_B)]} = 7.17$$
$$\overline{X}_B - \overline{X}_A \geq 55$$
$$\overline{X}_A - \overline{X}_B \leq -55$$
$$z_{\overline{X}_A - \overline{X}_B} = z_{-55} =$$
$$\frac{(\overline{X}_A - \overline{X}_B) - \mu_{\overline{X}_A - \overline{X}_B}}{\sigma_{\overline{X}_A - \overline{X}_B}} = \frac{-55 - (-50)}{7.71}$$
$$= -0.65$$

CASE OF B-A

$$\mu_A = 250, \sigma_A = 25, n_A = 30$$
$$\mu_B = 300, \sigma_B = 35, n_B = 40$$
$$\mu_{\overline{X}_B - \overline{X}_A} = \mu_B - \mu_A = +50$$
$$\sigma_{\overline{X}_B - \overline{X}_A} = \sqrt{[(\sigma_B^2 / n_B) + (\sigma_A^2 / n_A)]} = 7.17$$
$$\overline{X}_B - \overline{X}_A \geq 55$$
$$z_{\overline{X}_B - \overline{X}_A} = z_{55} =$$
$$\frac{(\overline{X}_B - \overline{X}_A) - \mu_{\overline{X}_B - \overline{X}_A}}{\sigma_{\overline{X}_B - \overline{X}_A}} = \frac{55 - 50}{7.71}$$
$$= +0.65$$

A drug was made as two tablet batches (A and B). **The average tablet weight of batch A is believed to be equal. to that of Batch B.** The standard deviations of tablet weight for the two batches are also believed to be equal and estimated as 21.214 mg. What is the probability that the difference between the means of two samples of sizes 9 to be at least 5 mg

Select one:

- ○ 0.7642
- ● 0.6892
- ○ 0.6171
- ○ 0.5485
- ○ 0.4839
- ○ 0.241
- ○ 0.105

A drug was made as two tablet batches (A and B). The average tablet weight of batch A is 530 mg and that of batch B is 570 mg. The standard deviations of the weight of the two batches are equal and estimated as 21.214 mg. The mean and standard deviation of the distribution for the difference between sample means based on B-A and sampling sizes of 9 are ------------------------------, respectively.

Select one:

- ○ -40, 10
- ○ 40, 5
- ○ 40, 100
- ○ -40, 100
- ● 40, 10
- ○ -40, 2.2
- ○ -40, 4.7

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